Learning to Model in Engineering

Julie Gainsburg

Published Version Information

Citation: Gainsburg, J. (2013) Learning to model in engineering.Mathematical Thinking and Learning, Volume 15, Issue 4,2013.

Digital Object Identifier (DOI): 10.1080/10986065.2013.830947

Learning to Model in Engineering

Abstract

Policymakers and education scholars recommend incorporating mathematical modeling into mathematics education. Limited implementation of modeling instruction in schools, however, has constrained research on how students learn to model, leaving unresolved debates about whether modeling should be reified and explicitly taught as a competence, whether it should be taught holistically or atomistically, and whether students' limited domain knowledge is a barrier to modeling. This study used the theoretical lens of *legitimate peripheral participation* to explore how learning about modeling unfolds in a community of practice—civil engineering—known to develop modeling expertise among its members. Twenty participants were selected to represent various stages of engineering education, from first-year undergraduates to veteran practitioners. The data, comprising interviews, "think-aloud" problem-solving sessions, and observations of engineering courses, were analyzed to produce a description of how this professional community organizes learning about mathematical models and resolves general debates about modeling education.

Keywords: Mathematical modeling, engineering education, reform mathematics

Policymakers and education scholars generally agree that mathematical modeling should be part of mathematics education at all levels (Blum, Galbraith, Henn, & Niss, 2007; Kaiser & Schwarz, 2006; Lesh, Hamilton, & Kaput, 2007). Modeling in some form appears in K-12 mathematics curriculum documents of many countries, notably Australia (Galbraith, 2007a), Canada (Suurtamm & Roulet, 2007), Denmark (Antonius, 2007), Germany (García, Maass, & Wake, 2010), the Netherlands (Vos, 2010), and the US (Common Core State Standards Initiative [CCSSI], 2010). Since 1983, the proceedings of the International Study Group for the Teaching of Mathematical Modeling and Applications have documented the design and evaluation of modeling activities or curricula for K-12 and university classrooms. Proponents claim that scholastic modeling has many benefits, including promoting deep understanding of mathematical and nonmathematical concepts (Lehrer, Schauble, Strom, & Pligge, 2001) and facilitating the transfer of mathematics to other domains (Cognition & Technology Group at Vanderbilt, 1990).

Unfortunately, the implementation of modeling in schools, especially primary schools (English, 2010), has been limited and inconsistent (Antonius, 2007; Blum et al., 2007; García, et al., 2010; Lehrer & Schauble, 2003), even in jurisdictions whose curriculum documents endorse modeling. This situation constrains research about how students learn to model and what instructional environments teach modeling most effectively. The literature offers a few broad theories about how students learn modeling (e.g., Haines & Crouch, 2007; Henning, & Keune, 2007; Lehrer & Schauble, 2003) and descriptions of students' responses to specific, local, model-teaching interventions (e.g., Crouch & Haines, 2007; Galbraith, Stillman, Brown, & Edwards, 2007), but a developmental view of learning to model has not been achieved (Blomhøj, 2011).

In the absence of longitudinal, articulated scholastic modeling programs in which to study the model-learning process, we might shift our research gaze to communities considered successful at building modeling expertise. This multi-case study investigated how learning about modeling proceeds in the civil-engineering profession. I took a naturalistic approach with a situated perspective (Greeno & MMAP, 1997), treating the setting as an object of inquiry as well as the behavior and learning it engendered. This approach mirrors Lave and Wenger's (1991) analyses of traditional and modern communities of practice (COPs) to understand how youth or novices are "apprenticed" into important cultural activity. Lave and Wenger sought a fresh perspective on the phenomenon of learning, unconstrained by its conventional association with formal schooling. Classroom experiments to teach and elicit modeling behavior have, likewise, been the main window on learning to model, but much should be gained by observing this phenomenon in a COP in which it has long been enculturated. Understanding how an engineering COP organizes learning about models could inform K-12 and university modeling education, especially if much of the learning were found to occur in the (college) classroom.

Theoretical Framework

This study was part of a larger project that examined the development of engineers' mathematical problem-solving abilities. In many STEM professions, mathematical modeling lies at the heart of problem solving. Thus, the problem-solving literature, which is far more extensive than that on modeling, offers a useful starting point for theoretically framing this investigation.

Problem Solving and Modeling

Definitions of "problems" converge on the idea that for something to be a problem, the solution path must not at first be readily apparent to the solver. The structural engineers I observed in prior research regularly encountered problems of this nature (Gainsburg, 2006, 2007a, 2007b). Due to the complexity and uniqueness of each project, established engineering theory and methods had to be adapted in ways not immediately evident, and for a few tasks I observed, no known procedures were even available. Complicating structural engineering is the fact that its objects—buildings and their behavior—do not yet exist, a fact that distinguishes the problem solving of engineers (and other designers) from that of scientists with access to empirical data for the phenomena they study. Engineering design is a bootstrapping process: The engineer starts with rough design assumptions, then analysis and design inform and refine each other through iterative cycles. Often, the goal of engineers' problem solving is a rational analytic method rather than specific design values (which are tentative through most of the project, anyway).

Recent reviews of research on mathematical problem solving (English & Sriraman, 2010; Lesh & Zawojewski, 2007; Lester & Kehle, 2003) portray modeling as central to problem solving in the modern workplace, as my prior research exemplified. Structural engineers must represent their design objects with mathematical (as well as drawn or physical) models in order to predict and study the objects' behavior. That is, engineering problems require models in order to be worked on, but selecting, adapting, or creating these models can itself be problematic. Descriptions of the cyclical mathematical-modeling process (e.g., Bissell & Dillon, 2000; Blomhøj & Højgaard, 2003; Lesh & Doerr, 2003) can be synthesized as follows:

1. Identify the real-world phenomenon
2. Simplify or idealize the phenomenon
3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)
5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

This process adequately describes the modeling of the engineers I observed as well as many modeling tasks proposed for K-12 education, though these can differ from each other. K-12 modeling tasks tend to involve generalizing from or fitting curves to data (given or student generated) (Galbraith, 2007b; Noss, Healy, & Hoyles, 1997; Radford, 2000). Structural engineers rarely engage in such activities, mainly because they lack initial access to data. Instead, their major challenges involve understanding structural phenomena deeply enough to simplify or idealize them for mathematization (Gainsburg, 2006). Lesh and Zawojewski's (2007) claim that "mathematical problem solving is about seeing (interpreting, describing, explaining) situations mathematically" (p. 782) applies well to structural engineering.

Reconceptualizing problem solving as "seeing" situations mathematically responds not only to the importance of modeling in today's workplace but also to the failure of prior problem-solving conceptualizations to guide instruction (Lesh & Zawojewski, 2007). Prior to the 1970s, experimental psychologists and, later, cognitive scientists saw problem-solving expertise as a set of cognitive processes invariant across domains. It could be studied in any context (including psychology labs) (Lester & Kehle, 2003) and attained by learning general problem-solving heuristics (e.g., those offered by Pôlya [1957]). The search for general heuristics, however, has not been fruitful, and more recent research reveals the domain-specificity of problem-solving expertise (English & Sriraman, 2010). Even within a domain, it is unclear how experts learn to solve problems (Lester & Kehle, 2003), and the conventional wisdom that experts first master domain knowledge, then learn strategies for selecting and applying that knowledge to problems, is now contested (Lesh & Zawojewski, 2007). The failure to identify global strategies and ways to promote problem-solving expertise has challenged the status of problem solving as a reified competency and instructional aim per se. Instead, a "models and modeling perspective" of problem solving (Lesh & Doerr, 2003) endorses the use of significant problems—in particular, mathematical-modeling activities—as vehicles for learning mathematical (and other) content.

Promoting scholastic modeling has not resolved the questions about problem-solving expertise, only transformed them into debates about the appropriateness of modeling education at particular grade levels, how explicitly to focus on modeling, and what pedagogical methods best promote modeling competency. For example, modeling requires deep knowledge about the phenomena to be modeled that students may lack (Blomhøj & Højgaard 2003; Simons, 1988). Because experts' knowledge is greater and better organized to support problem solving, teaching students to imitate experts' problem-solving strategies may be ineffective (Lesh & Zawojewski, 2007; Litzinger, Lattuca, Hadgraft, & Newstetter, 2011). Also, it may take years to master a mathematical skill sufficiently to apply it flexibly and fluently (Antonius, Haines, Jensen, & Niss, 2007; Dufresne, Mestre, Thaden-Koch, Gerace, & Leonard, 2005; Galbraith, et al., 2007), potentially compromising the effectiveness of modeling as a means to solidify newly learned mathematics concepts. These issues raise the question of the proper balance between (and sequence of) teaching about established models and having students develop their own (Schwartz, 2007). Also debated is the relative effectiveness of "atomistic" modeling instruction (teaching isolated modeling steps or heuristics) versus "holistic" (engagement in the full modeling cycle) (Blomhøj & Højgaard, 2003; Zawojewski & Lesh, 2003). A final question is whether modeling should be an explicit target of instruction (Julie, 2002) or a vehicle for learning mathematical (or other) concepts (Hamilton, Lesh, Lester, & Brilleslyper, 2008).

Modeling in Engineering Education

To date we lack a comprehensive picture of how engineers—or any other professionals—develop modeling expertise over their career. Crouch and Haines (2004) see modeling as a feature of undergraduate engineering instruction, but others (Carberry, McKenna, Linsenmeier,

& Cole, 2011; Whiteman & Nygren, 2000) disagree; this likely varies by engineering discipline as well as country.[1] Studies show engineering students, internationally, struggling to perform parts of the modeling cycle (Blomhøj & Højgaard 2003; Crouch & Haines, 2004; Soon, Lioe, & McInnes, 2011), but these studies offer snapshots in time, not developmental views. Undergraduates' limited engineering-domain knowledge is sometimes deemed an obstacle to modeling (Haines & Crouch, 2007), but little is known about whether and how the engineering community overcomes this obstacle. As in K-12 education, reformers call for increased modeling opportunities in engineering education, and the literature presents cases of engineering courses that integrate modeling (e.g., Clark, Shuman, & Besterfield-Sacre, 2010; Fang, 2011; Soon et al., 2011). Lesh and colleagues' program of "model-eliciting activities" (MEAs) may be the most thoroughly developed modeling initiative in engineering education (though the first MEAs were designed for K-12 students). In MEAs, small groups of students create, test, revise, and generalize mathematical models to solve realistic problems (Hamilton et al., 2008). MEAs engender a broad range of mathematizing processes, beyond curve fitting, so capture engineers' modeling behavior more authentically than do many school modeling tasks. Various studies document MEAs in engineering courses (Zawojewski, Diefus-Dux, & Bowman, 2008), but no study has evaluated their eventual impact on graduates' workplace performance (Hamilton, et al., 2008; Litzinger et al., 2011). Further, model-related engineering-education reforms are recent exceptions that cannot explain how today's veteran engineers attained their modeling expertise.

In this study, I sought to understand how the acquisition of modeling expertise unfolds over the course of the education of an engineer and how it is organized and supported by the COP. I also hoped to learn how the COP has resolved key questions about modeling education:

- To what extent is modeling reified and explicitly taught as a competence?

- Is modeling taught holistically (with novices employing the full modeling process) or atomistically (as separate steps and/or though heuristics)?
- Is limited domain knowledge seen as a barrier to teaching modeling and how is it overcome?

Learning to Model as Legitimate Peripheral Participation

This study takes the perspective of learning as *legitimate peripheral participation* (LPP) in a community of practice (COP) (Lave & Wenger, 1991). An LPP perspective seeks not to evaluate the way the COP produces experts but simply to describe how this occurs. The actions of novices and mentors are seen as facets of a single cultural process, with “learning” and “teaching” integrated and co-constituted. The activities, understandings, and perspectives of learners and mentors all provide valuable pieces of the picture of how a COP produces learning.

I argue that the engineering profession is a community of practice, one that includes engineering education. Professional associations, such as ABET (US and Canada) and the Engineering Council (UK), shape and monitor university programs and implement required licensing exams for engineers, and so constitute a major channel through which professional engineers take responsibility for educating novices. Other such channels include internships, the employment of practitioners as university instructors, and the formal and informal mentoring of new engineers in the workplace. An LPP perspective does not preclude examining formal instruction; it regards classrooms as sociocultural situations just as it regards informal, out-of-school settings (Greeno & MMAP, 1997). Indeed, engineering classes are sites for peripheral engineering activity that approximates professional tasks, for example, learning and practicing the algorithm for sizing beams. Once on the job, new engineers move from peripheral towards central activity, as they are first assigned small, standard parts of projects but gradually, over several years, take on larger, more complicated, less typical portions of projects.

Operationalizing Learning about Modeling in Engineering

LPP-oriented research focuses on the “opportunities [that] exist for knowing in practice” and “the process of transparency for newcomers” (Lave & Wenger, 1991, p. 123). For a study of learning to model, this means investigating novices’ opportunities to model and develop knowledge about modeling, how mentors make modeling practices transparent, what constitutes legitimate but peripheral modeling work, and how novices move from peripheral to central, expert performance. Observing a COP to learn how it builds expertise in a cognitive activity would be unproblematic if the COP had a shared, established understanding of the activity and deliberate, articulated ways of teaching it. Such is the case with procedural engineering skills like sizing columns or using CAD software. These are reified and universally built into coursework, and mentors know roughly what level of expertise to expect from new graduates in these skills. Mathematical modeling, as it turns out, is not this sort of activity. The veterans in this study did not share a definition of mathematical models, the university program had no deliberate plan to develop modeling expertise, and few students recognized modeling as a part of engineering they would need to learn. Thus, I had to make theoretical assumptions about the kinds of activities that would lead to modeling competence and the kinds of understandings that would prepare students to learn (Bransford & Schwartz, 1999) full-blown mathematical modeling activity.

I began with the six-step cyclical process to help me identify modeling activity. I sought evidence of novices performing or being asked to perform all or some steps of this cycle, as well as being given explicit instruction about how. Of particular interest was the mathematizing step (3). The literature offers no clear boundaries between mathematizing and developing mathematical models, but some common school tasks seem to qualify as the former only, such as algebraically representing phrases like “three less than a number” or calculating the area of a floor with multiple rectangular regions. Instructors may intend such tasks to be intermediate steps towards the more open-ended mathematizing required to model real phenomena, even if full-blown modeling is never attained in the curriculum (Højgaard, 2010). Thus, I sought evidence of opportunities for novices to apply mathematics in nonroutine ways, even if these were not situated in broader contexts that could be characterized as modeling. I even considered opportunities to estimate to be an entrée to modeling (Sriraman & Lesh, 2006). Evidence of novices performing all or some steps of the modeling cycle would address whether this COP took an atomistic or holistic approach to teaching modeling. In particular, opportunities to perform the mathematizing step (3), as well as interpreting (5) and testing (6), would illuminate how this COP construed and perhaps resolved the issue of limited domain knowledge.

I further presumed that an aspect of learning to model was an increasing awareness of mathematical models in coursework or practice. Here, I relied on my prior research about what expert structural engineers know and understand about models in their work (Gainsburg, 2006). Grounding their domain knowledge is the recognition that nearly all entities with which they work are models, not reality. They understand that models are built on assumptions, simplifications, idealizations, and tradeoffs, whose design consequences must be predicted. Thus, in the current study, I was attuned for evidence of instructors explicitly mentioning models, assumptions, and simplifications during lectures—as per LPP, making their knowledge about models transparent. I also listened for participants characterizing entities in their courses or work as models. My premise was that mathematical models permeated engineering-course content: Theoretical formulas model (and idealize) the behavior of physical elements. Drawings and diagrams also model (and simplify) elements and behavior, as do the setups of assigned exercises, which represent “real” problems with approximate values. Software also embeds models of physical elements or behavior. I was also attuned to participant remarks that key model-related concepts—assumption, idealization, simplification, iteration and revision, and uncertain, nonexistent, or inaccessible situations—were inherent in engineering. Even if a novice did not associate these concepts with models per se, I presumed this to be a developmental step towards full awareness of models. These observations would clarify the COP's stance on reifying and explicitly teaching about models, and further reveal an atomistic or holistic approach.

Methods

Research Tradition

This research was conducted as an instrumental, interpretive, sociological case study. *Instrumental case studies* focus on and describe a particular phenomenon or social process to illuminate understanding (Merriam, 1998; Stake, 2005). They do not claim to represent a broader population yet are expected to offer insights that may apply beyond the case. *Interpretive* case studies yield conceptual categories that illustrate theoretical assumptions (Merriam, 1998). *Sociological* studies focus not on an individual but on socialization (Merriam, 1998), here, via the lens of LPP. Although this study centered on one engineering program, my methods reflected a *multi-case study*. Participants were selected to represent distinct temporal stages of learning engineering (the cases), and analysis occurred on two levels: within case and then cross case (Merriam, 1998). This study also exemplified "tracing back" (Lofland & Lofland, 1995), starting with an outcome (the experienced modeler) and attempting to discern typical stages through which people pass towards that outcome.

Setting and Participants

Most study participants were connected to the civil-engineering program at my home institution, a large, public university in southern California. This choice offered maximum accessibility and a context with which I had some familiarity. California State University, Northridge admits students from the top third of their high-school classes, prioritizing local applicants. Roughly half of our students transfer in from two-year colleges. The undergraduate civil-engineering program, which graduates about 55 students annually, has undertaken no major reforms to the curriculum in recent years and so was expected to bear a general resemblance to other traditional programs at many US universities.

Participants were purposely sampled to support a “controlled comparison” (Maxwell, 1996) among six stage-cases spanning from the periphery to the center of engineering practice. I selected 20 participants: 2 students in each of Years 1 and 2 of the program, 3 students in each of Years 3 and 4, some with engineering work experience, 2 recent program graduates working as engineers, 4 instructors, and 4 veteran engineers (Table 1). The investigation of the schooling stages centered on four semester-long courses identified by program faculty as key to teaching the use of mathematics in engineering problem solving: Calculus II (Year 1, offered by the mathematics department), Statics (Year 2), Strength of Materials (Year 3), and Reinforced Concrete Design (Year 4) (these last three offered by the civil-engineering department). I targeted these courses for observations and soliciting student and instructor participants; during interviews, however, students and instructors reported on other courses and experiences.

I collected demographic information from the 19 students who volunteered for the study, including their age, years in the program, courses taken, and relevant work experience. I selected the 2-3 students who most closely resembled the typical student for that program year in terms of course history (had taken the expected prior courses but not advanced further), age (Year 1 students at age 18, etc.), and engineering work experience (none in Years 1 and 2; some for some of the Year 3 and 4 students). The two new engineers were identified through a participating instructor (few very recent program graduates were practicing engineers). Two veteran engineers were professional colleagues of another participating instructor and thus had ties to this program; the other two, from another local firm, were solicited through personal connections.

Data Collection

Interviews were the primary means of uncovering the participants' understandings about models. I interviewed the veteran engineers and instructors once and the students twice, early and late in the target course, to detect any changes in their understanding of or experiences with modeling that might have resulted from the semester's coursework. New engineers were also interviewed twice, several months apart. I designed different hour-long interview protocols for each participant type. The appendix shows specific questions from the longer protocols that yielded data for this study. I audiorecorded and transcribed all interviews (over 26 hours total).

After each interview, the students participated in an individual "think-aloud" problem-solving session, in which they worked on an assigned problem set from the target course and verbalized their thinking and strategies. With the instructors' assistance, I chose the first and last assignments in each course that typified course problem sets (e.g., as opposed to test review). After 45 minutes of work on a set, I stopped the student and asked additional interview questions (appendix). I audiorecorded these sessions (17 hours total) and kept copies of the students' written work. While transcribing the recordings, I also traced through each student's written work to understand every step. If the student had made an error in an intermediate step, I completed the solution using the incorrect value to see if he or she had proceeded correctly from there. I then supplemented the think-aloud transcript with explanations of the student's methods that I inferred from the written work.

I observed the two weeks of classes in Calculus II and Statics that preceded each think-aloud assignment; for Strength of Materials and Concrete Design, with which I was less familiar, I observed nearly every class. I documented these 75 total hours of classroom observation with fieldnotes and course-related artifacts (textbook pages, assignments, exams, etc.). My observations and fieldnotes took two perspectives: I attempted to capture explanations of problem-solving procedures from a learner's perspective, to enable me to understand the thinking that the students would exhibit during the think-aloud sessions. Then, with a researcher's perspective, I attended to instructor comments and behavior that potentially carried implied or explicit messages about the relationship between mathematics and engineering.

Certain methods afforded data triangulation, in particular for the schooling stages. I obtained the emic perspectives of students and instructors on each course, as well as student perspectives on the mathematics-engineering relationship and on modeling. These were complemented by more etic data: my own observations of instruction and of students working on engineering problems. New and veteran engineers also reflected on their education, which for some had been at Northridge, giving a retrospective view on the program. Veterans supplemented the new engineers' descriptions of on-the-job mentoring from the mentors' standpoint. The second interview of students and new engineers included many questions from the first interview. Although the primary purpose of this repetition was to detect growth in these participants' understanding, the similarity of their answers across interviews provided a degree of reliability. Additionally, several questions asked after the think-aloud sessions attempted to capture the same kinds of understandings as did certain interview questions but differently: Rather than asking about models or the mathematics-engineering relationship in the abstract, as in the interview, questions after the think-aloud sessions referred to the problems the student had just worked on or to recent lessons on that course material. I expected this contextualization to spark recognition of modeling or thoughts about how mathematics was used in engineering beyond what the students could articulate in response to abstract versions of the questions.

Data Analysis

Case-study data analysis typically aims for description and category construction, followed by categorical aggregation, from which patterns and correspondences among categories emerge (Creswell, 1998; Merriam, 1998). My general process was to start with *a priori* categories based on my operationalization of modeling behavior and understanding, as described earlier, then to distill patterns across the stage-cases that would indicate development.

Because I transcribed all interviews within days of their occurrence and wordprocessed my course-observation fieldnotes within hours of each class, data analysis began almost simultaneously with data collection. I kept a log of initial impressions from the interviews, think-alouds, and classes, noting ways participants portrayed or understood the relationship between mathematics and engineering. Within transcripts, I highlighted lines where models or related concepts were discussed; if this occurred in a class, I recorded the instance and date in my log.

These initial efforts led to more formal data categorization. In my first systematic analytic pass, I inspected all interview and think-aloud data for evidence of my *a priori* categories:

- Participants describing or characterizing mathematical models
- Opportunities for students or new engineers to perform all or parts of the modeling cycle
- Participants describing learning or teaching about mathematical models
- Participants characterizing engineering in terms of modeling subconcepts (assumption, idealization, simplification, iteration and revision, and uncertain, nonexistent, or inaccessible situations), regardless of whether they related these to modeling.

For each participant, I created a document of every excerpt from the interviews and think-aloud transcripts that related to each category. I then read the course-observation fieldnotes and artifacts for any instances fitting these same categories and added their description to the document associated with the course instructor. As the participants in this study occupied different positions in the engineering COP, it was inappropriate to combine the data for general categorization. My analysis needed to preserve distinctions among the groups. Further, because my objective was to uncover themes and broad patterns, not develop grounded theory or verify external theory, the relative frequencies of evidentiary statements and events were immaterial (Goetz & LeCompte, 1984). Thus, I could not use a formal coding scheme that required fracturing the text into discrete elements, regrouping, and enumerating them across contexts. Instead, I employed "contextualizing strategies" aimed at understanding all data in context and the relationships connecting those data into a coherent whole (Maxwell, 1996, p. 79).

Befitting a multi-case study, I performed within-case analysis prior to a cross-case one (Merriam, 1998). To accomplish this, I relied on summary writing as a main analytic tool (Lofland & Lofland, 1995; Merriam, 1998; Yin, 2003). From my first-pass analytic documents, I created separate narratives that described each category for each participant, capturing details from the data but adding a layer of interpretation to bring a broader picture into view (Goetz & LeCompte, 1984). From these individual narratives, I then synthesized a case-level narrative representing each group at each stage, highlighting patterns but retaining individual differences. Once these case-level narratives had been developed, I turned to cross-case analysis, to "build abstractions across cases" (Merriam, 1998, p. 195). I constructed summary narratives that reorganized the stage-cases, allowing naturally occurring themes to emerge. Closely related to the *a priori* data-analysis categories I had used for the individual and stage-level cases, these themes provided a meaningful structure for rich description of how these civil engineers learned to model and how this civil-engineering subcommunity organized this learning. I present summaries of these themed, cross-case narratives in the next section.

Learning to Model

Explicit Awareness of Mathematical Models

Students' awareness. I deliberately did not mention models during the first part of any interview, to see if participants would raise them spontaneously while answering questions about the relationship between mathematics and engineering. Only one student, Dmitri[3] (Year 4), did so, although two others, Ann (Year 1) and Claude (Year 3), uttered the word model when discussing software. Claude was also the only student to recall an instructor using the word model (once, also in a software context, by a computer instructor), but Connor (Year 3) ventured that his instructors might have been referring to models when they said "example" or "diagram."

Dmitri, with nine years of engineering-work experience and far more mathematical training than other students in the study, was the only student participant with a solid awareness of models. He first raised them while complaining about his high-school physics course:

> They assumed that the majority of the students taking that course were not going to have a math background, so they kind of edited the mathematical reasoning out of the course. And later on I realized the mathematical-model building was the core of what physics is. (Dmitri, Interview 1)

Dmitri did not attribute his awareness of models to specific courses. He felt he had always known that course exercises were "gross oversimplifications…or very special cases," particularly in earlier courses, where the models were simplest. His awareness of models had further developed at work, where "you get to see what engineers get away with, as far as simplifying assumptions." While aware of the role of models in engineering theory, Dmitri did not assign model creation to the workaday engineer. He referred to the models used in his work as "black-box algorithms…not directly accessible" to the engineers. Although I observed civil engineers creating mathematical models in my prior research (Gainsburg, 2006), Dmitri's observations may accurately have reflected practice in the tiny company in which he worked.

New engineers' awareness. The two new engineers, Nicki and Naomi, had more formal engineering education than Dmitri. They had earned the BS degree in civil engineering, Nicki was completing a masters program, and both were licensed engineers. Yet they were at an earlier stage in their awareness of models than Dmitri, likely due to fewer years in the workplace and less mathematical training. In her first interview, just days after she had rotated from project management into a new position in geotechnical design, Nicki did not believe she used mathematical models at work. Seven months later, she was able to identify only one example of a model in her work—a detail (drawing) of a retaining wall—but she struggled to articulate why this was a model and seemed unsure if it truly was. She did not recall undergraduate instructors using the term mathematical model but said it arose on occasion in her graduate classes:

> In the theory courses, yes. In the design courses not as much, because you're literally doing real-world problems. So as in the theory, you'll take a four-story building and it'll just be broken down into like mass mass mass mass, whereas in a design course, you're not breaking anything down. You're literally looking at the floor, looking at the beams, looking at the connections, at every tiny detail. [*Me: So you're not really modeling?*] Right. It's just exactly how it is. (Nicki, Interview 1)

Nicki seemed to deny the role of simplification in design, believing that simplification was only necessary for theoretical analysis. Her striking phrase "literally looking at the floor" failed to acknowledge the theoretical model mediating the translation from an actual (not yet existing) floor to its representation in a computer drawing.

Like Nicki, Naomi, in her first interview, did not recall having heard about models in her coursework, nor did she believe she used models at work, although she had heard coworkers talking about them. Only when, between interviews, she was assigned to a new project that explicitly involved mathematical models did she become aware of using them. Nevertheless, when I prompted Naomi to use this new awareness to reflect on whether her undergraduate courses had involved mathematical models, she continued to insist they had not.

Veterans' awareness and perspective on explicit teaching. No veteran practitioner spontaneously mentioned models when discussing the relationship between mathematics and engineering. Eventually, sometimes after much prompting, all four confirmed mathematical models were central to their work. Yet they varied in their views on the importance of modeling as part of engineering expertise and on the value of reifying the idea of models and calling attention to the fact that engineers used them. Vincent[4], the most skeptical on these points, said:

> It's not a word that I use…. It seems to be a jargon thing. You know, "I'm building a model; a mathematical model." That seems intimidating, as opposed to saying, "OK, here's a bunch of parameters; come up with a way to solve," and you realize you've just done a mathematical model. So I don't have that comfort level with that word.

The veterans also had difficulty articulating how they had become aware of mathematical models and their role in engineering, partly because their education had been decades ago. Only Vaughn thought mathematical models had been directly addressed in his schooling, but Vernon remembered a thermodynamics-exam item that he believed had been intended to teach modeling:

> Here's a hairdryer with so many watts of power; how long will it take you to dry your hair? So you have to guess how many hairs you have on your head, how long the hair is, how much water's in it, you know? It's just guessing, assuming things, and coming up with an answer.

Vernon also attributed his knowledge about models to "years of programming things" (such as mathematical representations of beams) into computer code, something he noted today's new engineers have less experience with, thanks to more sophisticated software. Vincent and Vlad felt they had been aware of working with models throughout their schooling but they did not recall having explicitly been taught this.

Instructors' perspective on explicit teaching. Unlike the veterans and most students, three of the four instructors spontaneously raised models in their interviews. All four instructors were as vague as the veterans about how they had learned about models. The Year 1 and 3 instructors did not directly credit school with this learning but saw it as a longitudinal process with many sources. The Year 2 instructor was certain that models had not been explicitly addressed in his schooling. Only the Year 4 instructor attributed any of his understanding of models to school but made clear this was graduate school. Perhaps due to their own educational histories, the instructors' approach to building undergraduates' awareness of mathematical models could be described as passive: assuming students would learn what they needed to know by the time they were practicing engineers but making limited or indirect efforts to promote that learning. Most interesting were their diverging opinions of the value of reifying models in education. The three engineering instructors acknowledged that models figured centrally in their courses, but they saw little reason to make this fact transparent before graduate school. In contrast, the Year 1 instructor, whose calculus students spanned a range of majors, believed modeling and being aware of models were important enough for a dedicated mathematics course:

> I wish [our mathematics department] emphasized modeling a whole lot more. I would like us to have maybe even a one-unit course in modeling. Choosing variables in an application problem, to finding them, and even labeling figures…. It would be a course in which they could maybe just use a lot of algebra. Maybe they wouldn't have had to have any calculus yet. But the point is not so much to use the algebra to solve the problems—that would be part of it—but it's setting up the problems…. If we had a separate course in it, it makes the point of how important just that phase of it is.

These varied opinions on teaching about modeling seemed due to idiosyncrasies of the instructors rather than to their station in the program, given that they each taught courses at all levels and were asked to take that broader perspective during their interview. Indeed, the Year 1 instructor was known in the mathematics department for comparatively heavy reliance on applications. Nevertheless, he confessed that he did not address models to his own satisfaction.

The Year 2 instructor represented the other extreme of the four. He suspected his students were unaware of what mathematical models were and that they were using them in their courses, but he did not consider this problematic. He saw little need to reify modeling for the beginning engineer so did not teach about models explicitly. He expected that *research* engineers, during their education, would come to understand that they were working with models because they would have to create them, but that workaday engineers might never come to this realization or need to. Even though the Statics curriculum and exercises nearly always invoked "real" elements and forces (e.g., a weight on a spring), the Year 2 instructor took a strongly mathematical approach in teaching. The word model nowhere appeared in his extensive handouts (essentially the course textbook), which described the given mathematical expressions and diagrams as “representing” or “indicating” structural phenomena, implying equivalence.

The Year 3 and 4 instructors shared some of the Year 2 instructor’s skepticism about explicit attention to models in undergraduate engineering courses, although they expected even non-research engineers to develop an awareness of models, in graduate school or on the job. Neither instructor claimed, nor was observed, to teach about models per se. The Year 3 instructor recognized that everything he taught was a model, but he did not believe his students were aware of it. Ultimately, he wanted program graduates, when given a problem, to “think about it and extract the key variables and construct a mathematical relationship between those variables that allows you to manipulate that system.” But he felt Year 3 students lacked the physical knowledge needed for modeling. The Year 4 instructor told me models were central to his courses, but when I asked if his students knew they were using models, his answer was qualified:

> I don’t think they will articulate the mathematical modeling concepts the way I am articulating it, but again, remember, I’ve been doing it for—But, yes, they do understand it. Maybe not at the same depth and maybe—Let me put it to you this way: They know that it’s a mathematical model, but maybe they don’t give it as much importance as I do, as an educator…. As they are progressing and they are seeing the applications over and over again, and they are seeing how the mathematical model is working, then, even in Statics, they know there is a mathematical model.

This seems optimistic, given the lack of explicit reference to models and the nature of assigned problems, which, as I discuss later, required only identifying and enacting taught procedures.

Summary. With one exception, the students in this study had virtually no awareness of mathematical models and the role they played in engineering courses or work, at least that they could articulate. Instructors and veterans confirmed the importance of mathematical models in engineering, but they generally saw little value in making them explicit in coursework or mentoring; they were confident that, with work experience, new engineers would develop the necessary awareness. One question this study sought to answer was the extent to which modeling is reified and explicitly taught as a competence. In this engineering subcommunity, mathematical modeling is considered an advanced concept, acquired as a byproduct of extended participation in engineering work rather than from direct, explicit instruction for novices.

Understandings about Mathematical Models

Students' understandings. When directly asked what the term mathematical model meant, all students except Dmitri had to guess. The guesses of the Year 1 and 2 students were vague and rambling and could incorporate several ideas, some of which changed between a student's two interviews or even within one. Two of these students made an association with physical models—Ann to a Rubik's Cube, as a model *of* mathematics, Ben to scale models—but they both suspected these associations were wrong. Ben's initial guess that a physical scale model was a mathematical model had come from a high-school assignment to build a "model" truss out of wooden sticks. Although he quickly dismissed this guess, he retained an idea that he abstracted from the truss example: A mathematical model was a "system." He elaborated:

> Like the truss; the whole setup of how each beam connects to it and the angles and everything, like, it's all connected as a whole. 'Cause I just think of models as, like, something you put together. (Ben, Interview 2)

Albert, Ann, and Ben ultimately guessed that a mathematical model was an equation or formula, Ann adding that it had to be exact (to explain why she did not consider $F \approx mg$, a formula given in her second think-aloud problem set, to be a model). Albert initially defined a model as a graph but later allowed that an equation could also be a model. Albert and Bradley explicitly linked mathematical models to real situations but, like Ann and Ben, they left open the possibility that a mathematical model could also model mathematics. Relatedly, three of these students guessed that a mathematical model was a process or algorithm for solving a problem, within or outside the domain of mathematics. Of the Year 1 and 2 students, only Bradley expressed the idea of a mathematical model as an idealization:

> I guess I'd say [a mathematical model] is probably kind of either ideal circumstances? Like, from an engineering standpoint, a lot of the things we do are trying to describe what's going on in a situation, but realistically there's a lot more forces that we just can't account for and a lot more going on than what we see. (Bradley, Interview 2)

During the first interview, no Year 3 student knew the meaning of a mathematical model. Connor would not even guess, and Chuck could only imagine that it might be a physical model or drawing with mathematics "put into it." Claude's first instinct was that a model was a standard: "Kind of like a standard that you can look at to see if your number's—your idea's within reason." They fared better in the second interview, when Chuck gave Mohr's Circle[5], the topic of his second think-aloud problem set, as an example of a mathematical model, because "it's a way to find out the numbers; it's a way to draw a circle and basically wherever everything is—So it's kind of like a convention." Later in this interview, Chuck broadened his notion of model to include "all the equations" from his courses, because they "explain how—when something is true." Connor, in his second interview, raised the idea of a solution process, guessing that a mathematical model might be "the mathematical steps to solving a problem." Claude, answering a question about technology in his second interview, referred to "modeling software" and a program in which "you can model circuits." Yet when asked immediately afterwards what the term mathematical model meant to him, he was still unsure, guessing that it was a physical representation of a building, or perhaps "a math concept without numbers." Suddenly, he recalled the software he had just described:

> Or modeling something into the software, like a model of a building or a house or something, or a beam. That would be, if you're modeling something, that's what—You're recreating it in the software. [*Me: So if that were a model, how would you define that kind of model?*] It's a representation of some physical—something that's real, in software. So that you can analyze it or just have plans.… You're modeling it into—so that the computer does the math for you and does all the analysis; you get, you know if it's gonna fail or not, whatever you're looking for. (Claude, Interview 2)

Claude then noted that Mohr's Circle was a mathematical model, because "the graph is describing, like, everything that you need to know about the solution of the problem."

Excluding Dmitri, the Year 4 students' understanding of models was no further developed than the Year 3 students'. Doug guessed a mathematical model was "a problem" in his first interview and "an example" in his second. Daniel guessed that a mathematical model was a graph that gave information about structural material or other physical conditions needed to solve a problem. In his first interview, he cited a stress-strain curve[6]; in his second interview, he listed an interaction diagram[7] as well as a rainfall-intensity chart from his hydrology class. Based on these examples, he concluded that a model was "something that you reference but that you would have to be able to solve some mathematical portion in order to use it." The graphs Daniel cited indeed reflect mathematical models, but his description contained neither the ideas of simplification and assumption nor an allowance for non-graphical models.

Dmitri had a well-developed understanding of mathematical models. When asked what the term meant to him, he cited several valid examples from the Year 4 course, then added:

> I guess the mathematical model is taking a problem or a type of problems and simply creating, usually, in most of these cases, a geometrical construct that most closely models or follows the shape of whatever it is that you're trying to find. And then by applying certain equations that have been empirically, or whatever, found to be characteristic of the materials, then finding out certain numerical properties of this figure that can be applied to the design process along with various sort of empirical fudge factors! (Dmitri, Interview 1)

In his second interview, Dmitri's definition of mathematical model highlighted assumptions. He gave the example, from the Strength of Materials class, which he had taken the year prior, of a formula for curvature in which one term was presumed small enough to ignore.

New engineers' understandings. In both interviews, Nicki groped her way to fairly valid definitions of a mathematical model but admitted she was guessing. In her first interview, she only offered, "To me it means taking something real world and breaking it down into parts that you need in order to solve a problem." In her second interview, she gave more detail:

> I would say [a model] is a problem that you're setting up to solve. At least in engineering, it would be like a free-body diagram? So you have a picture of a bunch of stuff and you're simplifying it to the point where you can just isolate all the forces and so that you can solve the problem…. [*Me: Would you just say a model is a simplification, or is that going too far? Is it not quite so general?*] I think, uh, a simplification in order to solve it. Like, a necessary simplification, I guess. (Nicki, Interview 2)

Thus, to Nicki, a mathematical model was a purposeful simplification, serving the process of solving the problem at hand. She could not think of an example beyond the free-body diagram until I pressed, when she proposed, “Any picture of a problem would be a mathematical model…because you have to have something to solve…. You have to draw it to get your mind focused on what the problem is.” This description suggests she saw mathematical models as translations; however, she never described a translation into mathematics, only into pictures. When asked if anything she did in AutoCAD or Excel struck her as a model, Nicki answered no,

> Unless you just consider an equation a mathematical model. [*Me: Would you?*] I don’t know! I don’t think I would, but I guess you could. Technically, any math would be a mathematical model maybe? I don’t know. (Nicki, Interview 2)

Naomi’s understanding of models in her first interview was similar to Nicki’s. Naomi was only able to say that a mathematical model was a way to get from givens to outcomes, from which you could draw conclusions. Between interviews, her employer, an environmental engineering firm, had hired a mathematics consultant to create projective models, and Naomi had been tasked with finding, verifying, and analyzing historical data to feed these models and helping interpret the numerical and graphical results. This required some understanding of the equations underlying the models, though she never wrote or modified the equations. Several sources had contributed to her learning about these models: prior reports, outside research, conferences, and mentoring. Returning for her second interview, she was excited to share her new understanding with me, although her explanation was fuzzy:

> A mathematical model is an equation or a set of equations that, um, they model, once you put certain—once you change the variables, it gives you a different perspective of how things will change. So, for example, the hydraulic models, those are mathematical models

> that you change the precipitation or temperature and then you see how the curve will go; if changing temperature will change how much precipitation you will get that year, you know?… They kind of show you the bigger picture at the end. Can be predictive. Can be for the present, to see if you have, for example, in a structure, and you can use a hydraulic—any mathematical model, to see how much load you can put on that structure before it fails. (Naomi, Interview 2)

Even with her new understanding, Naomi still maintained that none of her undergraduate courses had involved mathematical models, only “equations and concepts.” I asked her to distinguish equations she considered models from ones she did not:

> Well, the equations that they’re not hydraulic, those are set equations. You calculate, for example, discharge. You have a friction; you have velocity; you can calculate your discharge. That will not change. It doesn’t matter what kind of data you have; it will not change. However, the mathematical models are—You derive them from a set of data, so it differs from, it can be different depending on your graph, depending on your data. You derive it. You don’t have it; it’s not a set equation. I mean, the hydraulic models that I look at right now, they’re not going to be replicated in any other watershed because this is specific to—yeah. [*Me: So they’re not universal things? There aren’t universal physical models or engineering models?*] No. (Naomi, Interview 2)

Apparently, through this single work experience, Naomi had come to understand a model as a mathematical relationship designed to predict the behavior of a process. Other remarks during her interview revealed her grasp of the significance of initial assumptions: Models were “dumb,” she noted, and only reflected what you put in. But she believed models were designed for specific data sets and were not universal or general; they had to be homegrown, thus ruling out anything she saw in coursework and other workplace tasks as models.

Veterans' understandings. Three of the four veterans struggled to define mathematical model—Vincent even asked me what I meant—indicating this was not a term they were used to explaining. In lieu of definitions, they listed properties, examples, or ingredients of mathematical models. As ingredients, all mentioned equations or numbers, two mentioned boundary conditions or starting assumptions, one mentioned physical properties, and one mentioned an understanding of the situation. As to purpose, three veterans said models were for solving for or predicting behavior (but Vincent said engineering models were *not* for prediction), and Vlad said models stood in for phenomena that did not exist or were too big to bring into the office or classroom. Two said models were simplifications. The veterans also cited various instantiations of mathematical models at work: computer programs and spreadsheets, drawings, equations, and calculations. All four conflated mathematical and computer models during their interviews. Yet they even diverged on what they considered a computer model—either the general theoretical model underlying the software or the specific representation of the building; thus, two colleagues gave different answers to whether they created new models or used existing ones.

Instructors' understandings. The instructors gave richer descriptions of mathematical models than the veterans. As a group, the instructors named several purposes for models—to understand phenomena, see trends and relationships, and manipulate the represented system—and they listed multiple ingredients: physics, equations, functions, graphs, variables, constants, probabilities, geometric shapes, value tables, and vectors. Two instructors said models captured main features or variables; another mentioned that models captured *relationships among* variables. Three instructors called models simplifications, but one noted that models could become increasingly complicated during a project—the only reference by any participant to the iterative nature of models, perhaps because I had asked for the meaning of model, not model*ing*. Notably, no instructor mentioned computer models when describing mathematical models, in sharp contrast to the veterans, for whom these were essentially synonymous.

Another type of characterization concerned the relation of models to reality. The Year 1 instructor portrayed models as “wrong” but still useful, noting that they were not literal representations. Two instructors saw mathematical models as divorced from reality, at least temporarily. The Year 2 instructor explained

> [A mathematical model] is something that doesn't have to make sense…. For example, I put some number [in], and the result doesn’t really have to be a force, a positive or negative force; it’s just a number. So in other words, when I want to transform an engineering problem into a math, I probably just want to focus on: How do I solve the problem? Doesn’t matter if the result makes sense or not…. [*Me: When you say it doesn’t make sense, do you mean it doesn’t have a physical representation?*] Right, right!... For example, I’m calculating in vector algebra, but each of these vector terms does not have to carry any physical sense, like forces or velocity.

The Year 4 instructor, somewhat similarly, explained

> Once you accept the mathematical model, the rest follows suit mathematically.… We may argue, on a graduate level or esoterically, if the mathematical model is right or wrong, but once we accept a mathematical model, the rest just follows very deductive reasoning.

Both of these instructors remarked that after a solution was generated it had to be evaluated according to physical criteria. But both described a process whereby, once modeling conditions were set, the “real world” could be ignored and model solving could proceed entirely through calculation and deduction. This contrasts with portrayals of everyday mathematics (Carraher & Schliemann, 2002; Lave, 1988), as well as with my observations of structural engineers’ mathematical practice (Gainsburg, 2006), where intermediate steps maintained their contextual ties.

Summary. The students (excluding Dmitri) and, initially, the new engineers could only guess at the meaning of a mathematical model. Their varied guesses included a solution process, a standard or example, and a drawing or graph. Thanks to a workplace project that made modeling explicit, one new engineer had begun to grasp what models were and how they were used, but her understanding was underdeveloped and localized. The comments of the instructors and veterans reveal the absence of a community-established definition of mathematical models, even though these veterans performed expert-level modeling work every day. Articulating the meaning of models was not a straightforward task for these experts; their definitions were fragmented and diverse. In this light, it is unsurprising that their students and mentees had fragile conceptions of models. These findings offer further evidence that modeling was not reified and explicitly taught to novices in this subcommunity.

Modeling Subconcepts and Partial Understandings

Students’ partial understandings. While this engineering program did not explicitly address mathematical models, aspects of the program had potential to build model-related understandings that could prepare students to develop fuller knowledge about models on the job or in graduate school. Several student participants possessed concepts inherent to mathematical models—that engineering required simplification and idealization and that mathematics enabled performance predictions when physical testing was impossible—without connecting these ideas to models per se. Answering my question about why engineering students were required to learn so much mathematics, Ann drew on her automotive background to give an insightful rationale for mathematical models in engineering, without ever mentioning models:

> How are you gonna decide whether this steel is hard enough when someone hits it? It's not gonna crush your passenger? Can you do that right now? Without math? No. And the company's not going to give you, like, five million different sheets of metal so you can smash all of them. That's a waste of time. They're paying you to work. They want you calculate it. (Ann, Interview 1)

Ben discussed idealization in engineering, also without connecting it to models:

> In Statics right now, we're assuming that a beam will be able to take whatever load and that the beam doesn't bend at all, which in real life it will, and not every beam can take an unlimited amount of force. Which is kind of what we're learning now. We're learning ideal cases, which the world is not an ideal case all the time. (Ben, Interview 2)

And although he knew little about models, Daniel displayed some awareness that engineering representations were simplified for calculation purposes when he noted the similarity of the mathematical processes for beam and footing design: "Because of the footing, if you turn it upside down, it's kind of like a beam." He did not connect this observation to models.

Instructors' perspective on instilling partial understandings. Although the Year 3 and 4 instructors did not teach about models per se, both made overt and continual efforts to portray the uncertainty and subjectivity of engineering. These efforts seemed to be natural byproducts of these instructors' commitment to expose the realities of practice, rather than deliberate strategies to build an understanding of models. Both of these instructors concurrently worked as engineers, and they frequently referenced real situations and delineated practical implications of the methods or solutions they presented in class. Occasionally they set class problems in the context of a story from their professional work, to illustrate the influence of human desires or error.

The Year 3 instructor explained to me that his strategy for moving students towards understanding models was to start with simple problems and examples, then make them increasingly realistic and complex. This stated strategy did not address uncertainty, subjectivity, and model "fit," but in class I observed him articulating these conditions. When introducing new analytic methods—the bulk of his course—he would spend a few minutes on their mathematical and historical derivations, framing them as invented theories that "solved" long-standing engineering problems. (He did not refer to these "solutions" as mathematizations or models; indeed, I recorded his in-class use of the word model only once, in a context in which it meant "example.") In the process, he would explain what assumptions had to be made for the method to work and under what physical conditions those assumptions were "good" or "bad." He noted when aspects of formulas, primarily signs, were set by convention. Some of his in-class comments implied that engineers made subjective choices among methods to best approximate real behavior; a set of perfectly reflective formulas would never exist. None of his assignments, however, required a choice except of the most prescribed sort: If the value of *x* is greater than 5, use Formula 2. Only twice during class did he directly depict engineering's subjectivity, both times proclaiming engineering to be an art, not a science (once after showing how two methods yielded opposite predictions about an element failing). Other remarks he made in class implied that methods and formulas were chosen for efficiency, "because structural engineers are the laziest people in the world." In general, the Year 3 instructor's frequent comments of this nature, many of which seemed like asides, should have left his students with the impression that the analytic methods of the Materials course were human inventions that approximated empirical observations, and which would be refined over time as mathematics advanced.

The Year 4 instructor also grounded his teaching in practice. His Concrete course addressed element design, not just analysis, and he made daily references to construction, interpreting calculated results (e.g., element dimensions) in terms of feasibility and explaining how, in practice, they would be adjusted to reduce error or cost. He mentioned the word model only once in class, but he often invoked the subjectivity and uncertainty of engineering. He characterized the formulas and methods of concrete design and analysis as “not theoretical” but derived to fit experimental data, implying the human creation of engineering mathematics, and he warned that concrete capacities in the field would differ from the lab-generated values used in the course. He explained that more predictable loads could be assigned smaller safety factors, which would be further reduced as materials and science improved. He related how the Northridge earthquake had disproven analytic assumptions previously taken as “gospel.” Overall, despite the absence of explicit references to models, his course seemed likely to further the Year 4 students’ understanding of the subjective, uncertain, creative nature of engineering design and analysis.

I saw less evidence that the Year 1 and 2 courses would instill pre-modeling understandings, although the Year 1 instructor used many realistic problem situations and on occasion commented on the “fit” of certain graphs or equations to given data sets. Though most students probably left his calculus course ignorant of the meaning of the term model (as did Ann and Albert), they may have obtained the sense that people create mathematical equations to represent real situations and that these equations will always have inaccuracies.

Summary. The instructors in this program moved students towards legitimate modeling practice and understanding very gradually, by making transparent certain aspects of engineering work that embodied key modeling subconcepts. These subconcepts included uncertainty, subjectivity, idealization, assumption, and the importance of practical implications. Some students had come to recognize some of these subconcepts as central to engineering, without necessarily linking them to modeling. Addressing another research question, modeling was neither taught holistically nor via separate steps or heuristics in this subcommunity. Yet the instructors employed an approach that could be considered atomistic, building student understandings of key modeling subconcepts that, intentionally or not, could constitute important preparation for future learning about modeling in full.

Opportunities for Novices to Perform All or Some Steps of the Modeling Cycle

Students' modeling opportunities. Consistent with the lack of explicit attention to models in this program, I never observed students being asked to perform the full modeling cycle, nor did any student describe such opportunities in a course or workplace internship. Opportunities to engage in parts of the cycle were limited to Steps 4 and 5: performing the mathematical manipulations and (less often) interpreting the mathematical solution in real-world terms. Virtually all course exercises involved applying taught algorithms to straightforward situations. The assignment I used for the second Calculus II think-aloud set was exceptionally nonroutine and thus had been made extra credit. This single, multipart problem asked students to compare a "more accurate" gravity formula to an approximation given by the first two terms of a Taylor series. In effect, it asked students to compare two mathematical models for gravity, but it did not use this language. In the Year 2 and 3 courses, almost all exercises had realistic contexts (e.g., forces on beams) but none invoked practical considerations.

I was particularly interested in whether students were given structured opportunities to mathematize or taught how, invited to make novel applications of mathematics to real situations, or even to estimate. The think-aloud problem sets indicated that students were offered very little opportunity to mathematize. Except for the second Calculus II think-aloud problem, the mathematical methods required for every assigned problem were essentially pre-specified: All problems clearly matched, by diagram or topic, a method and example that had been presented in a recent lecture, and students almost always solved these problems by imitating the examples. When students could not identify the correct method or got stuck following it, they rarely attempted to solve it another way during our session. Instead, they abandoned the problem and planned to consult with friends or search online for the topic or even specific problem, confident of locating an established procedure. Apparently, their courses never required them to quantify real phenomena (e.g., by generating data) or formulate equations to answer nonmathematical questions about phenomena. All students agreed that homework and exam problems could almost always be solved by following procedures given in class, and the Year 3 and 4 students observed that, over the years, course exercises had only changed in topic and amount of parts.

Across the courses, students noted that homework and exam problems afforded only small degrees of flexibility, for example, different ways to set up a proportion or use trigonometry to solve a triangle. What students found most challenging were minor modifications to the in-class examples: A rectangular footing in the class example became a trapezoid on the homework; a solid element in class had a hole on the exam. These modifications came closer than anything else to requiring mathematization, albeit on a low level.

Specific interview questions probed the extent to which students viewed mathematics as a flexible tool, adapted mathematical methods, or invented their own when a situation exceeded what known methods covered. Most scoffed at the idea that, as students, they could or should invent mathematical methods, a position congruous with program expectations. Most students acknowledged that problems could have multiple solution routes, and some took advantage of this fact by avoiding recently taught methods and doggedly adhering to their first-learned, less-efficient, better-understood ones. Some students described mathematical shortcuts they had worked out, and many would work backwards from given solutions. A few related trying to re-derive methods during exams (when they had forgotten a procedure or missed a lecture) but confessed to a low success rate with this. Year 4 students were required to estimate initial dimensions to use in design algorithms, then interpret the results to refine their estimates. This process illustrated the iterative nature of design, but this iteration was never connected to modeling. Otherwise, students reported they were rarely expected to make estimations, though some claimed to estimate answers prior to solving as a correctness check. Those with work experience knew that professional engineers relied on estimates and accepted lower levels of precision than were expected in school; Dmitri called his boss's approximations "terrifying." All students struggled to imagine an engineering scenario where established mathematical methods would fall short. Only Daniel offered one: In his senior design course, he had to choose a design value less than a specified limit. He had been stymied without a procedure to guide this choice.

Instructors' perspective on modeling opportunities. The instructors confirmed that teaching students to mathematize was not a focus of this program. Asked whether he ever expected his students to invent mathematical methods, the Year 2 instructor explained:

> I don't want them to spend time on that. I want them to spend [time] on the basic mathematical applications, applying those basic things to the new subject that we learn and focus on the new subject.… In design classes, I know the professors might use

> estimations. But in my classes I don’t, because I have to have the analytic solutions. So in my classes we have to follow the math.

The Year 4 instructor acknowledged the importance of mathematical flexibility in engineering but downplayed the program’s role in developing it, due to the domain knowledge it required:

> For [students] to be able to go, in a sense, go “off the reservation,” figuratively, of course, needs a lot of practical experience.… The more they practice, the more they will realize, OK, these things I can get away with; those things I will not be able to get away with.

Veterans’ perspective on modeling opportunities. Echoing the instructors, the veterans believed their newest colleagues lacked the domain knowledge needed to model structures, to identify which aspects were important to include in models and which could be ignored, and to recognize when established models (e.g., beam formulas) could be used. Vernon and Vlad placed some blame on the overuse of computer models in engineering programs, which compromised what Vlad called “hands-on, practical, why does this work” knowledge. But all veterans felt sufficient domain knowledge for modeling could only be obtained through years of experience. They told a consistent story about how new colleagues learned about models once on the job, a story that encapsulated legitimate peripheral participation. As Vaughn explained:

> We try to start [new engineers] on small tasks and have them work alongside someone like [veteran] so that we can kind of develop a certain level of comfort with what people are doing and ease them into it. Rather than just give them a whole big modeling project. We just kind of try to do it in baby steps till we gain confidence that the person does really kind of understand basic statics, basic dynamics.

This strategy seemed to capture Nicki and Naomi's situations. Nicki was still being assigned small tasks (grading land) to build up her domain knowledge; Naomi had moved on to more central participation in modeling yet was still exposed only to certain aspects of the overall cycle.

Summary. Addressing my final research question, the instructors and veterans in this study agreed that the limited domain knowledge of students was a barrier to their ability to mathematize (and thus model) engineering phenomena. The route to learning to use or create mathematical models in this subcommunity was apparently not through significant course-based opportunities to make novel applications of general mathematics to real situations.

Discussion

To achieve adequate depth for a study of legitimate peripheral participation in a community of practice, only a small subcommunity can be observed. Taking that subcommunity to represent the greater COP must be done cautiously. While the civil-engineering program at Northridge is probably typical of those at large, regional US universities, the degree to which the responses of the participants in this study represent the experiences of students and instructors in other programs, or even in this one, is unknown. Indeed, the range of understandings of models in this local sample nearly ensures that an even wider range exists in the broader engineering community. Special caution is warranted in drawing conclusions about instructional "inputs." Although I asked participants to speak about in- and out-of-school experiences beyond the target courses, I did not directly observe those.[8]

Despite these caveats, I believe this study paints a clearer picture of the development of mathematical modeling in a profession than was previously available. Briefly, the mentors in this subcommunity consider undergraduates unready for explicit instruction in modeling, primarily because they lack the engineering-domain knowledge to support the decisions needed for modeling, reflecting the concerns of Blomhøj and Højgaard (2003), Schwartz (2007), and Simons (1988). Thus, instructors deliberately do not make modeling practice transparent, per se. They do, however, make transparent other aspects of engineering practice—simplifying, making assumptions, representing inaccessible phenomena in forms that can be quantified and analyzed, and making subjective judgments—that are important subconcepts of mathematical modeling, and students do pick these up. This constitutes a form of LPP that may prepare novices for future learning (Bransford & Schwartz, 1999) about full modeling as graduate students or professionals. On the job, legitimate participation becomes increasingly central, as mentors assign new engineers small, standard design and analysis tasks that mirror the straightforward mathematical procedures taught to undergraduates. Over the years, as new engineers build domain knowledge, collaborate with and are supervised by experts, and earn those experts' trust, they are assigned more complex projects requiring increasingly sophisticated modeling.

Before drawing implications, it behooves us to ask: Might civil-engineering mentors underestimate the potential of explicit modeling instruction? Might it be more feasible than they imagine to give novices access to legitimate engineering-modeling activity and knowledge? The theory of LPP allows that COPs do not always promote learning in the most efficient ways. Sometimes experts fail to make practices and knowledge transparent, and newcomers may be sequestered from legitimate participation (Lave & Wenger, 1991). My findings may imply inefficiencies in the learning system or unnecessarily restricted access for novices, in this subcommunity or the broader COP. For instance, although the experts in this study, like those in Gainsburg (2006), saw mathematical models as central to civil engineering, an understanding of what constituted a mathematical model did not appear to be industry established; it was largely tacit and sometimes confounded with a computer model. Some experts saw modeling as a process in which the engineer performed all steps of the modeling cycle; to others, modeling was a matter of entering values into software programs. Further, despite their awareness of the ubiquity of models in engineering, these experts had apparently spent little time considering the overall process of learning about models and modeling. Most believed that the necessary understanding of models would be attained, somehow, by the time it was needed at work, which could be a few years in. Unlike engineering education in general, which appears deliberately sequenced and designed to complement workplace learning, learning about models seems haphazard, unintentional, and under-theorized; the instructors and veterans in this study had no mental scheme for teaching about them. Because they believed insufficient engineering-domain knowledge precluded authentic modeling in the undergraduate curriculum, they sequestered the students from legitimate modeling activity. As a result, the students in this study possessed weak knowledge about mathematical models, probably even weaker than the veterans imagined. It was difficult for the students to obtain knowledge about models without explicit instruction. Even work experience that explicitly involved models, as Naomi received, did not reveal or teach every aspect of modeling; development in modeling expertise was slow even once on the job.

This picture presents possible leverage points for improving engineers' learning about models. The engineering community might benefit from discussions among experts, especially those in mentoring roles, to define a mathematical model in engineering (or per specific engineering discipline). Armed with a clear definition, mentors could be more explicit about modeling (Julie, 2002) and make their modeling behaviors transparent. Relatedly, agreement on key modeling subconcepts might encourage mentors to formally engage novices with these concepts, offering them legitimate, peripheral modeling experiences earlier in their careers.

On the other hand, we might respect the traditional wisdom of the community. After all, it is unclear that the current state of the engineering profession indicates a need for more modeling instruction. Most veterans have apparently learned to model with sufficient competence, buildings and bridges get built, and no international “crisis” has been declared, blaming faulty engineering on poor modeling education. The veterans in this study saw few shortcomings in their own training, and they doubted, as did the instructors, that engineering modeling could be taught in a valid way to their students. Taking, then, the view that this COP has evolved a productive system for educating novices about modeling, its stance on key modeling-education debates may be informative. First, that stance problematizes the press for modeling in mathematics education. In this subcommunity, explicit modeling instruction was considered unnecessary because modeling would be learned on the job, informally. Legitimate modeling was seen as developmentally inappropriate before graduate school or in the early years of an engineering career, due to the amount of domain knowledge it required. Further, while this study showed no evidence that novices received atomistic modeling instruction (teaching isolated steps and heuristics) (Blomhøj & Højgaard, 2003; Zawojewski & Lesh, 2003), novices were taught modeling subconcepts that could serve as a foundation for holistic modeling, supporting the notion that modeling can be learned “in pieces.”

Even if we trust this subcommunity’s skepticism, the prognosis for modeling instruction may not be entirely bleak. It is still possible that a more explicit but general understanding of mathematical models—what they are and how they are created and refined—would prepare engineering students to develop a deeper, more effective, and earlier understanding of modeling once on the job. More broadly, general education about models, at K-12 or undergraduate levels, might better prepare all students to understand and use models in future education and across careers. Determining this potential would require longitudinal, comparative studies with controlled educational interventions, tracking the high school or college performance of students who experienced modeling instruction in lower grades, and following into various kinds of workplaces graduates who experienced modeling instruction in college.

A final possibility is that, even if explicit instruction in modeling at the K-12 or undergraduate levels does not lead to better understanding of models in adult work, it might provide other benefits claimed by modeling-education proponents. Classroom modeling activities might promote a deeper understanding and better transfer of mathematical concepts or concepts in other domains (Cognition & Technology Group at Vanderbilt, 1990; Lehrer, et al., 2001), or foster higher-order thinking or communication skills (Lesh, Lester, & Hjalmarson, 2003), which might enhance eventual professional performance. Again, these benefits can only be determined with longitudinal, comparative, empirical experiments.

References

Antonius, S. (2007). Modelling based project examination. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 409-416). New York: Springer.

Antonius, S., Haines, C., Jensen, T. H., & Niss, M. (2007). Classroom activities and the teacher. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 295-308). New York: Springer.

Bissell, C., & Dillon, C. (2000). Telling tales: Models, stories, and meanings. *For the Learning of Mathematics, 20*(3), 3-11.

Blomhøj, M. (2011). Modelling competency: Teaching, learning, and assessing competencies—Overview. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modeling: ICTMA 14* (pp. 343-347). New York: Springer.

Blomhøj, M., & Højgaard, J. (2003). Developing mathematical modeling competence: Conceptual clarification and educational planning. *Teaching Mathematics and its Applications, 22*(3), 123-139.

Blum, W., Galbraith, P., Henn, H-W., & Niss, M. (2007). Preface. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. xi-xiv). New York: Springer.

Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 61-100). Washington, DC: American Education Research Association.

Carberry, A. R., McKenna, A. F., Linsenmeier, R. A., & Cole, J. (2011). *Exploring engineering* *students' conceptions of modeling*. *American Society for Engineering Education Annual Conference & Exposition*. Vancouver, BC, Canada.

Carraher, D. W., & Schliemann, A. D. (2002). Is everyday mathematics truly relevant to mathematics education? In M. E. Brenner & J. N. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom* (pp. 131–153). Reston, VA: NCTM.

Clark, R. M., Shuman, L. J., & Besterfield-Sacre, M. (2010). In-depth use of modeling in engineering coursework to enhance problems solving. In R.A. Lesh, P.L. Galbraith, C.R. Haines, & A. Harford, (Eds.), *Modeling students' mathematical modeling competencies (ICTMA13)* (pp. 173-188). New York: Springer.

Cognition & Technology Group at Vanderbilt, (1990). Anchored instruction and its relationship to situated cognition. *Educational Researcher, 19*(6), 2-10.

Common Core State Standards Initiative (CCSSI). (2010). Retrieved on November 22, 2011 from http://www.corestandards.org.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions.* Thousand Oaks, CA: Sage Publications.

Crouch, R. & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical model. *International Journal of Mathematical Education in Science and Technology, 35(*2), 197–206.

Crouch, R. & Haines, C. (2007). Exemplar models: Expert-novice student behaviors. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering, and economics* (pp. 90-100). Chichester, UK: Horwood Publishing.

Dufresne, R., Mestre, J. Thaden-Koch, T., Gerace, W., & Leonard, W. (2005). Knowledge representation and coordination in transfer. In J. Mestre (Ed.), *Transfer of learning: Research and perspectives* (155-215). Charlotte, NC: Information Age Publishing.

Dym, C. L. (1999). Learning engineering: Design, languages, and experiences. *Journal of Engineering Education, 88*(2), 145–148.

English, L. (2010). Modeling with complex data in the primary school. In R.A. Lesh, P.L. Galbraith, C.R. Haines, & A. Harford, (Eds.), *Modeling students' mathematical modeling competencies (ICTMA13)* (pp. 287-299). New York: Springer.

English. L., & Sriraman, B. (2010). Problem solving for the 21st century. In B. Sriraman & L. English (Eds.) (2010). *Theories of mathematics education: Seeking new frontiers* (pp. 263-90). Berlin/Heidelberg: Springer-Verlag.

Fang, N. (2011). Students' problem solving in an introductory engineering course. *Proceedings of the 41stASEE/IEEE Frontiers in Education Conference*, Rapid City, SD. Retrieved on October 29, 2012 from http://fie-conference.org/fie2011/wsdindex.html.

Gainsburg, J. (2006). The mathematical modeling of structural engineers. *Mathematical Thinking and Learning, 8*(1), 3-36.

Gainsburg, J. (2007a). The mathematical disposition of structural engineers. *Journal for Research in Mathematics Education, 38*(5), 477-506.

Gainsburg, J. (2007b). Problem solving and learning in everyday structural engineering work. In R. A. Lesh, E. Hamilton, & J. Kaput (Eds.), *Foundations for the future in mathematics education* (pp. 37-56). Mahwah, NJ: Lawrence Erlbaum Associates.

Galbraith, P. (2007a). Beyond the low hanging fruit. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 79-88). New York: Springer.

Galbraith, P. (2007b). Dreaming a ‘possible dream’: More windmills to conquer. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering, and economics* (pp. 44-62). Chichester, UK: Horwood Publishing.

Galbraith, P., Stillman, G., Brown, J., & Edwards, I. (2007). Facilitating middle secondary modelling competencies. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering, and economics* (pp. 130-140). Chichester, UK: Horwood Publishing.

García, F. J., Maass, K., & Wake, G. (2010). Theory meets practice: Working pragmatically within different cultures and traditions. In R.A. Lesh, P.L. Galbraith, C.R. Haines, & A. Harford, (Eds.), *Modeling students’ mathematical modeling competencies (ICTMA13)* (pp. 445-457). New York: Springer.

Goetz, J. P., & LeCompte, M. D. (1984). *Ethnography and qualitative design in educational research.* Orlando, FL: Academic Press, Inc.

Greeno, J. G., & MMAP. (1997). Theories of practices of thinking and learning to think. *American Journal of Education, 106*(1), 85-126.

Haines, C., & Crouch, R. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 417-424). New York: Springer.

Hamilton, E., Lesh, R., Lester, F., & Brilleslyper, M. (2008). Model-eliciting activities (MEAs) as a bridge between engineering education research and mathematics education research. *Advances in Engineering Education, 1*(2). Retrieved October 29, 2012 from http://advances.asee.org/vol01/issue02/index.cfm.

Henning, H., & Keune, M. (2007). Levels of modeling competencies. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 225-232). New York: Springer.

Højgaard, T. (2010). Communication: The essential difference between mathematical modeling and problem solving. In R.A. Lesh, P.L. Galbraith, C.R. Haines, & A. Harford, (Eds.), *Modeling students' mathematical modeling competencies (ICTMA13)* (pp. 255-264). New York: Springer.

Julie, C. (2002). Making relevance relevant in mathematics teaching education. In I. Vakalis, D. Hughes Hallett, D. Quinney, & C. Kourouniotis (Compilers). *Proceedings of 2nd International Conference on the Teaching of Mathematics: [ICTM-2]*. New York: Wiley.

Kaiser, G., & Schwarz, B. (2006). Mathematical modelling as bridge between school and university. *Zentralblatt für Didaktik der Mathematik, 38*(2), 196-208.

Lave, J. (1988). *Cognition in practice: Mind, mathematics, and culture in everyday life.* New York: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Lehrer R., & Schauble, L. (2003). Origins and evolution of model-based reasoning in mathematics and science. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 59-70). Mahwah, NJ: Lawrence Erlbaum Associates.

Lehrer, R., Schauble, L., Strom, D., & Pligge, M. (2001). Similarity of form and substance: From inscriptions to models. In D. Klahr & S. Carver (Eds.), *Cognition and instruction: 25 years of progress* (pp. 39-74). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Hamilton, E., & Kaput, J. (2007). Directions for future research. In R. Lesh, E. Hamilton, & J. Kaput, J. (Eds.), *Foundations for the future in mathematics education* (pp. 449-453). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Lester, F. K., Jr., & Hjalmarson, M. (2003). A models and modeling perspective on metacognitive functioning in everyday situations where problem solvers develop mathematical constructs. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 383-403). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester, Jr. (Ed), *Second handbook of research on mathematics teaching and learning* (pp. 763-804). Charlotte, NC: Information Age Publishing.

Lester, F. K. Jr., & Kehle, P. E. (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 501-517). Mahwah, NJ: Lawrence Erlbaum Associates.

Litzinger, T. A., Lattuca, L. R., Hadgraft, R. G., & Newstetter, W. C. (2011). Engineering education and the development of expertise. *Journal of Engineering Education, 100*(1), 123-150.

Lofland, J., & Lofland, L. H. (1995). *Analyzing social settings: A guide to qualitative observation and analysis* (3rd ed.). Belmont, CA: Wadsworth Publishing Company.

Maxwell, J. A. (1996). *Qualitative research design: an interpretive approach.* Thousand Oaks, CA: Sage Publications.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass Publishers.

Noss, R., Healy, L., & Hoyles, C. (1997). The construction of mathematical meanings: Connecting the visual with the symbolic. *Educational Studies in Mathematics, 33,* 203–233.

Pôlya, G. (1957). *How to solve it* (2nd ed.). Princeton, NJ: Princeton University Press.

Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis. *Educational Studies in Mathematics, 42*, 237–268.

Schwartz, J. L. (2007). Models, simulations, and exploratory environments: A tentative taxonomy. In R. Lesh, E. Hamilton, & J. Kaput, J. (Eds.), *Foundations for the future in mathematics education* (pp. 161-171). Mahwah, NJ: Lawrence Erlbaum Associates.

Simons, F. (1988). Teaching first-year students. In A. G. Howson, J-P. Kahane, P. Lauginie, & E. de Turckheim (Eds.), *Mathematics as a service subject* (pp. 35-44). Cambridge University Press: Cambridge, UK.

Spandaw, J. (2011). Practical knowledge of research mathematicians, scientists, and engineers about the teaching of modeling. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modeling: ICTMA 14* (pp. 679-688). New York: Springer.

Soon, W., Lioe, L. T., & McInnes, B. (2011).

undergraduates in learning mathematical modelling. *International Journal of Mathematical Education in Science & Technology*, *42*(8),1023-1039.

Sriraman, B, & Lesh, R. (2006). Modeling conceptions revisited. *Zentralblatt für Didaktik der Mathematik, 38*(3), 247-254.

Stake, R. E. (2005). Qualitative case studies. In N, K. Denzin & Y. S. Lincoln, *The Sage handbook of qualitative research* (3rd ed.). (pp. 443-466). Thousand Oaks, CA: Sage Publications.

Suurtaam, C., & Roulet, G. (2007). Modelling in Ontario: Success in moving along the continuum. In W. Blum, P. L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 491-496). New York: Springer.

Vos, P. (2007). The Dutch maths curriculum: 25 years of modelling. In R.A. Lesh, P.L. Galbraith, C.R. Haines, & A. Harford, (Eds.), *Modelling students' mathematical modelling competencies (ICTMA13)* (pp. 611-620). New York: Springer.

Whiteman, W. E., & Nygren, K. P. (2000). Achieving the right balance: Properly integrating mathematical software packages into engineering education. *Journal of Engineering Education*, *89*(3), 331-339.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage Publications.

Zawojewski, J. S., Diefus-Dux, H. A., & Bowman, K. J. (Eds.). (2008). *Models and modeling in engineering education: Designing experiences for all students*. Rotterdam: Sense Publishers.

Zawojewski, J. S., & Lesh, R. (2003). A models and modeling perspective on problem solving.

In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 317-336). Mahwah, NJ: Lawrence Erlbaum Associates.

Footnotes

[1] Though perhaps not within countries. Dym (1999) claims US engineering programs "look far more alike than not," even across widely differing campus types.

[2] Undergraduates study engineering content for about 3,600 hours, whereas 10,000 hours of "active engagement in a domain" are thought necessary to attain expertise (Litzinger, et al., 2011).

[3] Year 1 students have been assigned pseudonyms beginning with A, Year 2 with B, etc. New engineers' pseudonyms begin with N and veteran engineers' with V.

[4] To protect the identity of the very few females in my sample of veteran engineers and instructors, I have assigned the male gender to all participants in these categories. Students' and new engineers' genders have been preserved.

[5] A graphical representation of the state of stress on a point on a body; the *x*-coordinates represent normal stress and the *y*-coordinates shear stress.

[6] A graph of stress (internal force per unit area) versus strain (deformation per unit length) for a particular material.

[7] A graph relating the axial loads and moments that produce failure in a structural column.

[8] Spandaw (2011), however, found similar attitudes towards college-level modeling instruction among Dutch mathematics, science, and engineering professors.

Appendix
Interview Questions Yielding Information about Modeling

From Student Interview 1
3) If a high school student asked you why they make you take a lot of math to be an engineer, how would you explain this? Do what extent do you agree with this?

6) Has your understanding of what math is changed any since coming to college? What do you think accounts for that change?

8) What kinds of technology are you being taught to use in your college math and engineering education? How does this technology simplify or complicate the math you do?

9) If I drew a continuum, from “Math is mainly a school subject” to “Math is everywhere in life” where would you place yourself to best represent your perception of math? Why? Do you think you are on a different point on this continuum than you would have been in high school, or earlier in this program?

12) What does the term “mathematical model” mean to you? Where did you learn this meaning? Has it meant different things to you at earlier points in your education?

13) Do you ever find yourself inventing mathematical or computation methods when solving engineering problems? If so, can you describe an example?

14) Do you ever find yourself rejecting mathematical methods for solving engineering problems in favor of some other way of reaching a solution, or estimating rather than calculating? If so, can you give an example?

From Student Interview 2 (*in addition to updated questions from Interview 1*)
3) Have the kinds of problems you are asked to solve this semester in you engineering courses changed any? Have they become harder or easier? In what ways?

4) Have you changed as an engineering problem solver this semester? What have you learned this semester about approaching engineering problems? What helped you learn that?

12*)* Have you ever been in an engineering problem solving situation where mathematical rules or formulas fell short and you had to resort to some other means to solve? Seen this at work?

17) Can you predict how problem solving on an engineering job will differ from what you do in classes today?

For Students After the “Think-Aloud” Sessions
11) Did any of these problems require you to use what you think of as a mathematical model?

13) How do the problems you solve for this course differ from problems you solved in high school math classes? Do they differ from problems you’ve solved for other college math or

engineering courses you've taken in prior semesters? Do they differ from problems assigned in courses you're taking now?

14) Thinking about yourself in earlier engineering courses [or earlier in this semester], have you changed as a problem solver? Do you approach problems differently? Do you have new problem solving "tools" that you didn't before? What do you think caused this change?

From New Engineer Interview 1

4) If a college freshman asked you why they make you take a lot of math to be an engineer, how would you explain this? Do what extent do you agree with this?

5) Did your understanding of what math was change in any way during college? Has it changed since starting this job?

7) How would you describe the relationship between math and engineering in your everyday work? Was this what you expected based on what you learned in college?

8) In what ways did your college courses prepare you to solve the engineering problems you now solve at work? In what ways could you have been better prepared?

9) Thinking back to the problems you solved in your engineering courses, how were those similar to and different from the ones you solve for work today? What did you find most difficult or challenging about those school problems? What do you find most difficult or challenging about the problems you solve for work now?

10) What kinds of math do you typically use today when doing your work? When faced with a problem or task, how do you know what mathematics to use?

11) Describe a current task you are working on and what math you are using. How did you know to use this mathematical method?

14) What does the term "mathematical model" mean to you? Has it meant different things to you at earlier points in your education or career? How did you learn about mathematical models?

15) What kinds of mathematical models do you use in your work? How often can you use already established models? Give examples. How often to do you have to modify models? Give examples. How often do you have to develop your own model? Given an example.

16) When you think about the most experienced engineers in this company, what do they know or what are they able to do that you don't yet know or can't yet do?

17) To what degree would you call their expertise mathematical? Do they seem to have a different kind of ability to apply math than you do?

18) So far, how have you learned how to apply math on the job? In what ways do colleagues or supervisors teach you this? In what ways do you learn it on your own? What do you think you

will have to do to over the years to acquire the expertise of the experienced engineers in your company?

19) Have you ever seen your veteran colleagues use their judgment to override or bypass what mathematical models or methods are telling them? Have you ever done this yourself? Give examples. To what degree do you see learning when *not* to rely on a precise mathematical solution part of engineering expertise? If you agree, how do you think you will learn this over the years?

20) What kinds of technology were you taught to use in your college math and engineering education? How were these similar to or different from the technologies you use in your work today? In what ways did college prepare you to use the technology in this workplace? In what ways could you have been better prepared?

21) How does technology simplify or complicate the math you do for work?

From New Engineer Interview 2 (*in addition to updated questions from Interview 1*)
1) What has developed in your personal work situation since we last spoke? Official changes? In what ways has the work you do every day changed since you began at this job?

16) Have you learned ways to use math to solve problems on the job that you didn't know when you started? How have you learned this? In what ways do colleagues or supervisors teach you this? In what ways do you learn it on your own? What do you think you will have to do to over the years to acquire the expertise of the experienced engineers in your company?

From Instructor Interview (*slightly modified for the Year 1 Calculus II instructor*)
2) If a college freshman asked you why they make you take a lot of math to be an engineer, how would you explain this?

3) How would you describe the relationship between math and engineering? Has your understanding of this relationship changed any since you became a professor?

4) How would you describe the mathematical abilities and dispositions of the freshmen entering your program? I'm particularly interested in their ability/disposition to use math in the service of solving engineering problems. How would you describe the mathematical abilities and dispositions of the typical graduate of your program?

5) My experience with high school students, even ones who are strong in math, is that they tend to see math as a school-only enterprise and seem to have limited ability to apply it to solving real problems. My prior research showed me that, in contrast, experienced engineers are flexible, fluent, and artful users of math in the service of solving everyday engineering problems. How do you think this mathematical transition happens? Is the ability to use math to solve engineering problems largely gained in university training? Or does it mostly happen once the engineer enters the workplace? What accounts for this learning, wherever it happens?

6) What personal efforts do you make in your teaching to help develop engineering students' ability and disposition to use math in the service of solving engineering problems? What other efforts are made (by faculty or via other experiences for students) during the engineering program to accomplish this transition?

7) What seem to be the major challenges for students when learning to solve engineering problems? What seem to be the major challenges for students when learning *to use math* in solving engineering problems? What seems to help them overcome these challenges?

8) Are there ways you think your program could better help develop students' ability and disposition to use math in the service of solving engineering problems? In an ideal world, what would be added, subtracted, or changed in the program to better accomplish this goal?

10) What does the term “mathematical model” mean to you? Has it meant different things to you at earlier points in your education and your career?

11) How central are mathematical models to your teaching? What do you hope your students will understand and be able to do regarding mathematical models at the end of your courses or at the end of the program? How do you help them reach that understanding? What other experiences in this program help them develop expertise with models?

12) Do you ever expect or hope that students will invent mathematical or computation methods when solving assigned problems? If so, can you describe an example?

13) Do you ever assign problems that require or allow students to reject mathematical methods in favor of some other way of reaching a solution, or require them to estimate rather than calculate? If so, can you describe an example?

14) How do you (or how does this program) help students transition from the kind of problem solving done in early courses: following examples and given procedures, to solving the kind of novel or ill-structured problems they'll see at work?

From Veteran Engineer Interview

4) If a college freshman asked you why they make you take a lot of math to be an engineer, how would you explain this? Do what extent do you agree with this?

5) Has your understanding of what math is changed since you first became an engineer?

7) How would you describe the relationship between math and engineering in your everyday work? Was this what you expected based on what you learned in your university program?

8) In what ways did your university training prepare you to solve the engineering problems you now solve at work? In what ways could you have been better prepared? Have other formal professional development experiences contributed to your learning how to solve everyday engineering problems?

9) Thinking back to the problems you solved in your engineering courses, how were those similar to and different from the ones you solve for work today? Has the nature of the problems you solve at work changed in your time at this job? What do you find most difficult or challenging about the problems you solve for work now? Has that changed?

10) What kinds of math do you typically use today when doing your work? When faced with a problem or task, how do you know what mathematics to use?

11) Do the computer-technology advances in your field simplify or complicate the math you do for work? How?

12) Describe a current task you are working on and what math you are using. How did you know to use this mathematical method?

13) Over the years at this job, have you changed in your ability to apply math to engineering problems? Has the way you apply math changed? What has brought about the changes in your ability and in your methods?

16) What does the term “mathematical model” mean to you? Has it meant different things to you at earlier points in your education and your career?

17) What kinds of mathematical models do you use in your work? How often can you use already established models? How often to do you have to modify models? How often do you have to develop your own model?

18) When you compare a veteran engineer to a brand new engineer in this company, what does the veteran know or is able to do that the newest engineer doesn’t yet know or can’t yet do?

19) To what degree would you call the veteran’s expertise mathematical? Do veterans seem to have a different kind of ability to apply math than the brand-new engineer?

20) How do new engineers learn how to apply math on the job? In what ways do colleagues or supervisors teach this? In what ways does the new engineer learn it on his/her own? What do new engineers in this company have to do to over the years acquire the expertise of the experienced engineer?

22) Sometimes engineers have to use their judgment to override or bypass what mathematical models or methods are telling them. To what extent do you agree that learning when not to rely on a precise mathematical solution is part of engineering expertise? How do you think new engineers learn this over the years?

Table 1

Participants

Status	Pseudonym[2]	Target Course	Engineering Work Experience	Other Notes
Year 1 Students	Ann	Calculus II	None	
	Albert	Calculus II	None	
Year 2 Students	Ben	Statics	None	
	Bradley	Statics	None	
Year 3 Students	Connor	Strength of Materials	None	
	Chuck	Strength of Materials	None	
	Claude	Strength of Materials	2 yrs (struct.)	Add'l yrs at construction and architecture firms
Year 4 Students	Doug	Concrete Design	None	
	Daniel	Concrete Design	3 yrs (land devel.)	
	Dmitri	Concrete Design	9 yrs (civil)	Extensive math training
New Engineers	Naomi	(NA)	1 yr (environ.)	
	Nicki	(NA)	2 yrs (civil)	New to geotech. design; Completing Civil Eng. MS program
Veteran Engineers	Vaughn	(NA)	20+ yrs (struct.)	Same large firm
	Vernon	(NA)	30+ yrs (struct.)	
	Vincent	(NA)	25+ yrs (struct.)	Principals at same small

	Vlad	(NA)	25+ yrs (struct.)	firm
Instructors	Year 1	Calculus II	None	Mathematics Dept.
	Year 2	Statics	2 yrs (struct.); add’l yrs in eng. research	Civil Eng. Dept.
	Year 3	Strength of Materials	30+ yrs (struct.)	Civil Eng. Dept.
	Year 4	Concrete Design	20+ yrs (civil)	Civil Eng. Dept.